

Understanding Heavy Quarkonium Systems in Perturbative QCD^{*†}

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We review the recent theoretical progress in heavy quarkonium spectroscopy within the boundstate theory based on perturbative QCD. New microscopic pictures of the heavy quarkonium systems are obtained.

1. Introduction

Recent advent of the boundstate theory based on perturbative QCD enabled accurate descriptions of the nature of the heavy quarkonium systems such as bottomonium states. Through the progress became more and more clear the importance to eliminate properly contributions of infrared (IR) degrees of freedom in the theoretical descriptions of these systems. Indeed these systems comprise natural IR cutoffs both of *spacial* and *temporal* dimensions. As a result, once we decouple the IR contributions properly, we find much better convergence of perturbative expansions and a good control over the theoretical predictions. In order to realize IR decoupling in a systematic way, the language of renormalons and their cancellations have played central roles: uncertainties of the perturbative expansions are estimated from their asymptotic behaviors based on renormalon dominance picture.

In the heavy quarkonium systems, we may classify the mechanisms of IR decoupling into two categories: (I) The spacial size of a quarkonium state acts as an IR cutoff. This leads to cancellation of $\mathcal{O}(\Lambda)$ renormalons, with residual $\mathcal{O}(\Lambda^3)$ renormalons. (II) Offshellness of the heavy quark and antiquark acts as an IR cutoff in temporal dimension. This leads to cancellation of $\mathcal{O}(\Lambda^3)$ renormalons, with residual $\mathcal{O}(\Lambda^4)$ renormalons. These aspects point to new and detailed physical pictures of the heavy quarkonium states.

2. Cancellation of $\mathcal{O}(\Lambda)$ Renormalons

Cancellation of $\mathcal{O}(\Lambda)$ renormalons [1] is a realization of the fact that the system under consideration is color-singlet and has a spacial size R much smaller than the typical hadron size $\Lambda R \ll 1$.

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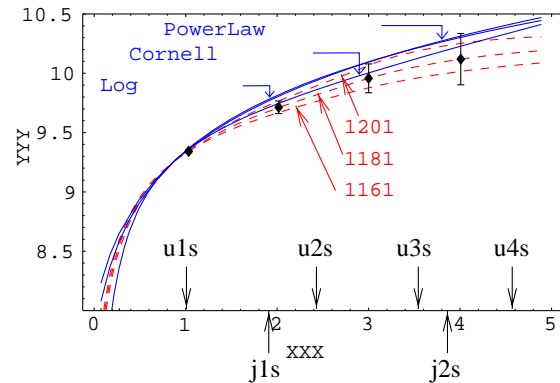


Fig.1: $E_{\text{tot}}(r)$ and typical phenomenological potentials. Constants are added to make all curves coincide at $r = 1 \text{ GeV}^{-1}$. Arrows at the bottom show the r.m.s. radii of the heavy quarkonium states. The figure is taken from [2].

2.1. Total energy of a static $b\bar{b}$ pair

Let us first neglect the kinetic energy of b and \bar{b} and examine the total energy of a static $b\bar{b}$ pair. It is defined as the sum of the static QCD potential and the pole masses of b and \bar{b} :

$$E_{\text{tot}}(r) = 2m_{b,\text{pole}} + V_{\text{QCD}}(r). \quad (1)$$

The $\mathcal{O}(\Lambda)$ renormalons contained in the pole mass and the QCD potential are cancelled if we rewrite the pole mass in terms of the $\overline{\text{MS}}$ mass, $\bar{m}_b \equiv m_b^{\overline{\text{MS}}}(m_b^{\overline{\text{MS}}})$. We thus substitute $m_{b,\text{pole}} = \bar{m}_b(1 + c_1\alpha_S + c_2\alpha_S^2 + c_3\alpha_S^3)$ to eq.(1) and expand $E_{\text{tot}}(r; \bar{m}_b, \alpha_S(\mu), \mu)$ in $\alpha_S(\mu)$ up to $\mathcal{O}(\alpha_S^3)$. Then the perturbative expansion becomes much more convergent as well as much less dependent on the renormalization scale μ (see eqs.(3)–(6) below). The remaining renormalon is of order $\Lambda \times (\Lambda \cdot r)^2$. Fig.1 shows $E_{\text{tot}}(r)$ corresponding to the present values of the strong coupling constant. They agree well with typical potentials used in phenomenological model analyses within the uncertainty estimated from the remaining $\Lambda^3 r^2$ renormalon (indicated by error bars), in the range relevant to the bottomonium and charmonium states.

Recently several comparisons have been made [3,4,2,5] among the perturbative predictions of the QCD potential (in renormalon-subtracted schemes), the QCD potential calculated by lattice simulations, and the phenomenological potentials in this region. Combining these analyses, we find that all these potentials agree well with one another. It appears that the non-perturbative effects, if they exist, are comparable in size to the perturbative uncertainty (apart from those in the r -independent part of the potentials, which has not been constrained by these analyses).

Phenomenological potentials in the above range may be represented by a Coulomb-plus-linear potential, which becomes steeper than the Coulomb potential at larger distances. We can understand why $E_{\text{tot}}(r)$ also becomes steeper than the Coulomb potential in perturbative QCD [4]. After realizing that the $\mathcal{O}(\Lambda)$ renormalon should be cancelled, it is natural to define a strong coupling constant $\alpha_F(\mu)$ from the interquark force:

$$F(r) \equiv -\frac{d}{dr} V_{\text{QCD}}(r) \equiv -C_F \frac{\alpha_F(1/r)}{r^2}. \quad (2)$$

Since the $\mathcal{O}(\Lambda)$ renormalon in $V_{\text{QCD}}(r)$ is r -independent, it is *killed* upon differentiation. $\alpha_F(1/r)$ grows at IR due to the running (the first two coefficients of the β -function are universal), which makes $|F(r)|$ stronger than the Coulomb force at large distances. This means that $V_{\text{QCD}}(r)$, after subtraction of the renormalon, becomes steeper than the Coulomb potential.

2.2. Bottomonium spectrum

The bottomonium energy levels can be computed in series expansions in α_S within the boundstate theory based on perturbative QCD. Presently the full corrections up to $\mathcal{O}(\alpha_S^4 m_b)$ are known [6]. It is illuminating to compare the series expansions before and after the cancellation of the $\mathcal{O}(\Lambda)$ renormalons. For instance:

• $\Upsilon(1S)$:³

$$M_{\Upsilon(1S)} = 9.94 - 0.17 - 0.20 - 0.30 \text{ GeV} \quad (3)$$

$$= 8.41 + 0.84 + 0.20 + 0.013 \text{ GeV} \quad (4)$$

• $\Upsilon(2S)$:⁴

$$M_{\Upsilon(2S)} = 9.94 - 0.10 - 0.19 - 0.45 \text{ GeV} \quad (5)$$

$$= 8.41 + 1.46 + 0.093 + 0.009 \text{ GeV} \quad (6)$$

³The parameters are taken as $\mu = 2.49 \text{ GeV}$, $\alpha_S(\mu) = 0.274$, $\bar{m}_b = 4.20 \text{ GeV}$, $m_{b,\text{pole}} = 4.97 \text{ GeV}$.

⁴The parameters are taken as $\mu = 1.09 \text{ GeV}$, $\alpha_S(\mu) = 0.433$, $\bar{m}_b = 4.20 \text{ GeV}$, $m_{b,\text{pole}} = 4.97 \text{ GeV}$.

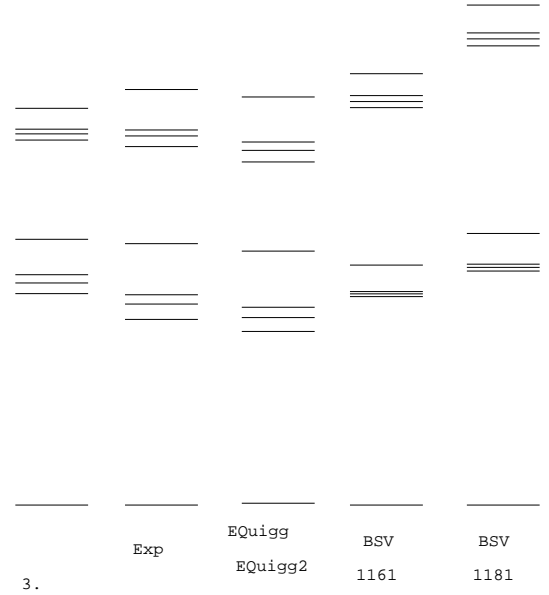


Fig.2: Bottomonium spectrum up to $n = 3$ as calculated in various frameworks compared to the experimental data.

The upper lines correspond to the pole-mass scheme and the lower lines to the $\overline{\text{MS}}$ -mass scheme. As can be seen, the convergence property improves dramatically when the $\overline{\text{MS}}$ mass is used instead of the pole mass. In this scheme, the perturbative series for the bottomonium energy levels show converging behaviors up to the states with the principal quantum number $n = 3$ [7,8].

Fig.2 compares the bottomonium spectrum calculated in various frameworks with the experimental data: BSV(1161) and (1181) represent the fixed-order perturbative QCD predictions up to $\mathcal{O}(\alpha_S^4 m_b)$ for the input $\alpha_S(M_Z) = 0.1161$ and 0.1181 , respectively [8]; RS(1181) represents the perturbative QCD prediction which includes in addition part of the higher-order corrections, in particular the full $\mathcal{O}(\alpha_S^5 m_b)$ corrections to the fine structure, for $\alpha_S(M_Z) = 0.1181$ and $\mu = 3 \text{ GeV}$ [9]; Eichten-Quigg represents the prediction of the phenomenological potential model [10]. The level of agreement between the perturbative prediction (RS) and the experimental data approximates to that of the Eichten-Quigg model.

2.3. Physical picture [7]

When the $\mathcal{O}(\Lambda)$ renormalons are cancelled, the major part of the energy of a bottomonium state

can be written as

$$E \approx 2\overline{m}_b + \int_{R^{-1} \lesssim q < \overline{m}_b} \frac{d^3\vec{q}}{(2\pi)^3} C_F \frac{4\pi\alpha_S(q)}{q^2}, \quad (7)$$

where R represents the spacial size of the boundstate. It shows that the energy is mainly composed of (i) the $\overline{\text{MS}}$ masses of b and \bar{b} , and (ii) the self-energies of b and \bar{b} originating from gluons whose wavelengths are shorter than the size of the boundstate $1/\overline{m}_b \lesssim \lambda \lesssim R$. As expected, contributions of IR gluons ($\lambda > R$) have decoupled from eq.(7). The potential energy between b and \bar{b} turns out to be much smaller than the self-energies (ii). This observation is reminiscent of the constituent quark mass picture for explaining the masses of light hadrons, if we regard the energy (ii) as the difference between (state-dependent) constituent quark masses and current quark masses.

In Fig.2 the level spacings between consecutive n 's are almost constant, in contrast to the Coulomb spectrum in which the spacing decreases rapidly with n as $1/n^2$. This is because the second term of eq.(7) grows rapidly as the size of the boundstate increases. Namely, the rapid growth of $\alpha_S(q)$ at $q \sim R^{-1}$ as R increases pushes up the energy levels of the excited states. This gives a microscopic description for how the interquark force between a color-singlet heavy quark pair becomes strong at large distances (within the range where the perturbative prediction is still valid). It is the rapid growth of the self-energies (ii) rather than of the potential energy that gives the dominant effect.

3. Cancellation of $\mathcal{O}(\Lambda^3)$ Renormalons

Let us briefly state a historical background related to this subject. As already noted, the energy of a static $b\bar{b}$ pair, eq.(1), contains $\mathcal{O}(\Lambda^3 r^2)$ renormalon, where we may replace r by the typical size $R \sim (\alpha_S m_b)^{-1}$ of the boundstate. Within the potential-NRQCD framework, it was shown [11] that this renormalon can be absorbed into a non-local gluon condensate, i.e. the perturbative uncertainty can be factorized and replaced by a non-perturbative parameter of the same dimension $\sim \Lambda^3$. On the other hand, it has been known for a long time [12] that the leading non-perturbative corrections to the quarkonium energy levels are $\mathcal{O}(\Lambda^4)$ since they are proportional to the local gluon condensate $\langle G_{\mu\nu} G^{\mu\nu} \rangle$. Thus, there is an apparent mismatch in the power of Λ

between the two quantities.

It has recently been shown [13] that, if the off-shellness of b and \bar{b} is incorporated properly, it provides an additional suppression factor of order $\Lambda/(\alpha_S^2 m_b)$ to the renormalons in the quarkonium energy levels:

$$\delta E \sim \Lambda \times \frac{\Lambda^2}{(\alpha_S m_b)^2} \times \boxed{\frac{\Lambda}{\alpha_S^2 m_b}}. \quad (8)$$

Hence, the dimension of the perturbative uncertainty becomes the same as that of the leading non-perturbative corrections (including the powers of α_S and m_b in the denominator). Moreover, convergence of the perturbative series improves by this effect.

Intuitively the suppression mechanism may be understood as follows. Large offshellness of b and \bar{b} corresponds to short rescattering time of b and \bar{b} inside the boundstate. If the rescattering time $\Delta t \sim (\alpha_S^2 m_b)^{-1}$ is shorter than the hadronization time $\sim \Lambda^{-1}$, b and \bar{b} will get distorted before IR gluons surround them and an energy is accumulated by antiscreening effects. Thus, we expect the offshellness to act as an infrared cutoff to the effects induced by gluons' timelike propagation.

In the (leading) kinematical configuration relevant to formation of the bottomonium states, gluons exchanged between b and \bar{b} have momenta q^μ where $|q^0| \ll |\vec{q}|$. Therefore, in the conventional approach we take the instantaneous gluon-exchange as the leading-order and incorporate perturbations to it. On the other hand, a different kinematical region, $-q^2 \sim \Lambda^2$, contributes to renormalons. Because of this, in order to estimate renormalons in the bottomonium spectrum accurately, we need to make an approximation valid in both of the above kinematical regions.

An analysis following this observation reveals that there is a rich mathematical structure underlying the renormalon suppression effect. For instance, the expansion about the instantaneous potential changes the analyticity of the Borel transform $\tilde{E}[u]$ of the bottomonium energy level. IR renormalons, defined as poles of $\tilde{E}[u]$ in the u -plane, are located at $u = 2, 3, 4, \dots$. But once the expansion is made, poles are generated at $u = \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \dots$, while at the leading-order of the expansion, the original poles at $u = 2, 3, 4, \dots$ disappear. For example, at $u = \frac{3}{2}$, there is a cancellation of poles in the combination

$$\tilde{E}[u] \sim \frac{\Delta^{3-2u} - 1}{u - 3/2}, \quad \text{where } \Delta = \frac{1}{2} C_F \alpha_S. \quad (9)$$

Δ represents the offshellness or non-instantaneity of the potential. However, if Δ is considered to be small and an expansion $\Delta^{3-2u} = \Delta^3 \times (1 - 2u \log \Delta + \dots)$ is performed, there remains an uncancelled pole at $u = \frac{3}{2}$. Hence, the perturbative uncertainty becomes larger by this expansion.

4. Conclusions

There has been important progress in the theory of heavy quarkonium states, which extended the predictive power of perturbative QCD beyond what could be achieved before. Perturbative predictions can be made accurate once IR degrees of freedom are eliminated properly from the calculations. Renormalon dominance picture suggests that uncertainties of the perturbative prediction for the bottomonium spectrum are of moderate size, of $\mathcal{O}(\Lambda^3/(\alpha_S^2 m_b^2))$ if the short-distance mass is used, of $\mathcal{O}(\Lambda^4/(\alpha_S^4 m_b^3))$ if the offshell effects are further incorporated. Evidence has been found for the static QCD potential and the bottomonium spectrum, which supports the hypothesis that the perturbative predictions agree with the full QCD predictions within the estimated moderate uncertainties.

Based on the perturbative QCD predictions, we obtained new microscopic pictures on the composition of the heavy quarkonium spectrum and on the behavior of the interquark force in the intermediate distance region. Furthermore, the quark offshell effect as an IR cutoff in timelike processes was recognized, whose nature is essentially non-local in time.

So far, most important applications of the progress are the precise determinations of the $\overline{\text{MS}}$ masses of b and c quarks: e.g. $\overline{m}_b = 4190 \pm 32$ MeV, $\overline{m}_c = 1237 \pm 60$ MeV have been obtained based on the above hypothesis that non-perturbative corrections can be absorbed into perturbative uncertainties [8].

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